Physics GRE Notes

Alex Wen

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This is a set of "last minute notes" I made for myself which helped me study for the exam. It probably has everything you need to remember, and assumes that you've already familiarized yourself with most of the concepts. I found it helpful to review this daily for a few weeks before the (Spring 2021) exam. Leafing through these also kept me nervously occupied while waiting for the test center to open...

If you find any mistakes here, I'd be grateful to know at alex.wen@alumni.ubc.ca.

Much of the structure and topic selection for these notes are based on *Conquering the Physics GRE* by Kahn and Anderson, and I highly recommend their newest version as a complete in-depth study guide.

Contents

1 Classical Mechanics

This stuff will probably be most universally familiar to physics students regardless of their program or school, and is accordingly heavily tested on the GRE. In my experience, mechanics questions, owing to their conceptual straightforward-ness, usually involve more calculation, numbers, and work than, say, atomic physics which in contrast involves more conceptual knowledge-recall questions.

I found little reason to add a lot of explanation, so this section is very brief with bare-bones equations and ideas. Personally, I did not struggle much with the difficulty or method to solve mechanics questions, but ran into annoyances making silly mistakes. Make sure to work through mechanics questions carefully.

1.1 Newtonian Stuff

There's not much to say here. Just know how to draw free body diagrams, balance forces, and $\vec{F}_{net} = m\vec{a}$. There will certainly be a few block-on-a-ramp type questions.

1.2 Energetics

Force is related to potential energy

$$\Delta U = -\int_a^b \vec{F} \cdot d\vec{l} \tag{1}$$

$$F = -\nabla U. \tag{2}$$

Work done by a force will be

$$W = \int \vec{F} \cdot d\vec{l} \tag{3}$$

but far more useful for the GRE just remember $W = F \cdot d$.

Remember the scalar forms of angular momentum and angular force (torque)

$$L = I\omega \tag{4}$$

$$\tau = \frac{dL}{dt} = I\alpha \tag{5}$$

(the vector form isn't really a GRE consideration).

A rotating reference frame will experience fictitious forces. *Inside* the rotating frame rotating at Ω you experience an additional coriolis force

$$F_{cor} \propto m\dot{r}\Omega \tag{6}$$

and a centrifugal force

$$F_{cen} \propto mr\Omega^2. \tag{7}$$

Technically there is a cross product but it seems to be unnecessary for the GRE.

The center of mass along any coordinate is

$$r_{CM} = \frac{\int r \, dm}{M} = \frac{\int \rho r \, dV}{\int \rho \, dV}. \tag{8}$$

1.3   Lagrangian and Hamiltonian Mechanics

The Lagrangian for a system is the kinetic minus potential energy

$$\mathcal{L} = T - V \tag{9}$$

and satisfies the Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \frac{\partial \mathcal{L}}{dq} \tag{10}$$

for each coordinate q. The *conjugate momentum* is $p = \frac{\partial \mathcal{L}}{\partial \dot{q}}$. If $\mathcal{L}$ (or $\mathcal{H}$) does not explicitly contain q, then $\frac{\partial \mathcal{L}}{dq} = 0$ and the conjugate momentum is conserved (unchanging in time).

The Hamiltonian for a system without explicit velocity or time dependence is

$$\mathcal{H} = T + V, \tag{11}$$

the only case you need to consider for the GRE. Also,

$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q} \qquad \dot{q} = \frac{\partial \mathcal{H}}{\partial p}. \tag{12}$$

Beware of the signs!

In general, the GRE will often give questions asking for the form of $\mathcal{L}$ or regarding conjugate momenta. It will probably not ask for long and involved calculations to solve the E-L equations for the equations of motion. You will often see $\mathcal{L}$ or $\mathcal{H}$ written in terms of p's. For example, a linear kinetic energy term can be $\frac{1}{2}mv^2 = \frac{p^2}{2m}$.

1.4 Orbits

The $\mathcal{L}$ of an orbit can be written in terms of polar coordinates.

$$\mathcal{L} = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2 - U(r) = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mv^2 - U(r) = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} - U(r) \quad (13)$$

Almost always we will have a radial potential $U(r)$. The angular momentum l is

$$l = I\omega = mr^2\dot{\phi}. \quad (14)$$

Naturally, the total energy of the orbit is

$$E = T + V = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r). \quad (15)$$

When $E > 0$, the orbit is unbound, and the trajectory is hyperbolic. When $E = 0$, the orbit is parabolic. When $E < 0$ the orbit is bound and elliptical.

For two masses orbiting each other, use the reduced mass to get the orbit of one mass in the frame of the other.

$$\mu = \frac{m_1 m_2}{m_1 + m_2}.$$

Kepler's three laws: first, orbits of planets are ellipses with the sun at one focus. Second, equal areas of orbit are swept out in equal times. Third, the period is related to the semi-major axis:

$$T^2 \propto a^3. \quad (16)$$

1.5 Harmonic Oscillators

The most general equation of motion is

$$m\ddot{x} + b\dot{x} + kx = 0. \quad (17)$$

There's the $m\ddot{x}$ term describing acceleration, the $b\dot{x}$ damping term, and the kx spring force term. Without the damping term ($b = 0$), the solution is a simple sinusoid. The solutions to the

characteristic equation are

$$r_{\pm} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = -\beta \pm \sqrt{\beta^2 - \omega_0^2} \tag{18}$$

where we've made the common identifications $\beta = b/2m$ and $\omega_0 = \sqrt{k/m}$. There are three classes of solutions:

1. $\beta^2 < \omega_0^2$, underdamped. Solutions will be of the form $x = Ae^{-\beta t} \cos\left(\sqrt{\omega_0^2 - \beta^2}t - \delta\right)$.

2. $\beta^2 = \omega_0^2$, critically damped. Solutions will be of the form $x = Ae^{-\beta t} + Bte^{-\beta t}$.

3. $\beta^2 > \omega_0^2$, overdamped. Solutions will be of the form $x = e^{-\beta t}\left(Ae^{t\sqrt{\beta^2 - \omega_0^2}} + Be^{-t\sqrt{\beta^2 - \omega_0^2}}\right)$.
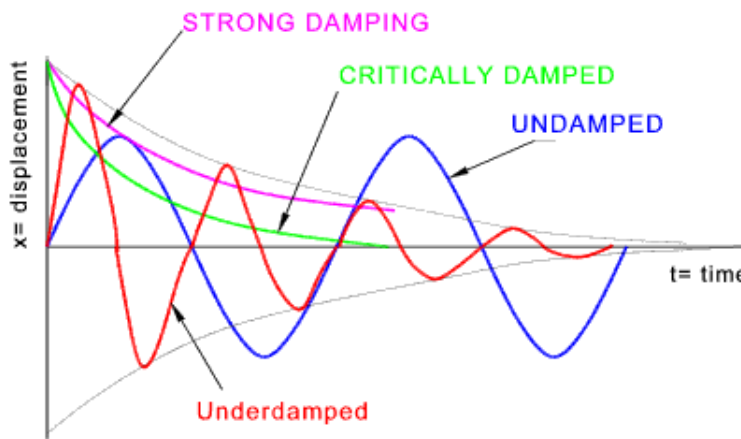
For classical oscillators it's unlikely that you'll be asked detailed questions or asked to solve for solutions. You'll most likely be directly asked qualitative questions or presented with very simple equations or constants (like β, ω_0, etc.) and then asked about qualitative characteristics.

Some quick words about forced (driven) motion. Systems will have a resonant frequency

$$\omega_R = \sqrt{\omega_0^2 - 2\beta^2}.$$

If the driving frequency approaches this resonant frequency, the amplitude of the oscillations increases rapidly. Note that in the absence of damping ($\beta = 0$), $\omega_R = \omega_0$.

Another easy pointer: coupled springs. For springs with k_1 and k_2 in series, the effective spring constant of the system k_{eff} is given by

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}. \tag{19}$$

For springs in parallel,

$$k_{eff} = k_1 + k_2. \tag{20}$$

Finally, normal modes and coupled oscillators are *technically* testable concepts, but I personally have seen minimal questions related to them. This might have to do with the complexity of these topics and the difficulty of creating suitable short GRE-style multiple choice questions. They are probably not worth worrying about, unless you have some spare time.

1.6 Fluids

Just know two things.

First, for an incompressible fluid (all you will consider for the GRE)

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant} \tag{21}$$

along the pipe (or streamline). v is velocity, g is gravity, z is height, p is pressire, ρ is density.

Second, bouyancy: the buoyant force is the weight of displaced water volume V:

$$F_{buoy} = \rho V g. \tag{22}$$

Don't forget that water has $\rho = 1 \text{ kg/L}$.

2 Electromagnetism

In my experience, some questions in this section feel like mechanics (straightforward approach), but some will start to test your conceptual understanding more deeply.

On the exam, I often did not pause at all (to think) when doing classical mechanics, but for E&M I sometimes stared at a question for a short while before knowing how to solve it. For this chapter especially, I recommend doing more than just memorizing formulas, and try to do as many practice problems as possible to improve on versatile problem-solving ability.

2.1 Review with Maxwell's Equations

Being intimiately familiar with Maxwell's equations will pay off, because they effectively summarize all the basic laws of electromagnetism. Equivalently, a good understanding of all electromagnetism will allow you to quickly reproduce Maxwell's equations.

1. Take the divergence of E. *The divergence of the electric field is proportional to the charge density.*

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}.$$

   Divergences are begging to be integrated over a volume, with surface S.

$$\iiint \nabla \cdot E \, dV = \frac{1}{\epsilon_0} \iiint \rho \, dV$$
$$\iint_S E \cdot dS = \frac{Q_{enc}}{\epsilon_0}$$

   On the LHS we applied the divergence theorem, giving us the flux through the surface. On the RHS, integrating a charge density over volume gives us the enclosed charge in that volume. Thus the *electric flux through a closed surface is proportional to enclosed charge (Gauss' Law).*

2. Take the curl of E. *The curl of the electric field is related to the change in magnetic field* (think of charge-carrying wires).

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

   Let's integrate both sides over any surface with closed boundary C.

$$\iint_S \nabla \times E \cdot dS = -\iint_S \frac{\partial B}{\partial t} \cdot dS = -\frac{\partial}{\partial t} \iint_S B \cdot dS$$
$$\int_C E \cdot dl = -\frac{\partial}{\partial t} \Phi_B.$$

   On the LHS we applied Stokes' theorem, and on the RHS the area integral of the field gives the magnetic flux. Note that the LHS is also the expression for voltage/EMF. The

voltage around a closed loop is the negative change in magnetic flux through the loop (Faraday & Lenz's Laws).

3. Take the divergence of B. *The divergence of the magnetic field is always zero.*

$$\nabla \cdot B = 0.$$

Integrating over a volume and applying the divergence theorem, like before with the E field, we get

$$\iiint \nabla \cdot B \, dV = 0$$

$$\iint_S B \cdot dS = 0.$$

The magnetic flux through any closed surface is zero - there exist no magnetic monopoles. There is no such thing as "magnetic charge" (that we currently know of)!

4. Finally, take the curl of B. *The curl of the magnetic field is related to the current density and change in electric field.*

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Again, like with the E field, integrate both sides over any surface with closed boundary C.

$$\iint_S \nabla \times B \cdot dS = \mu_0 \iint J \cdot dS + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \iint E \cdot dS$$

$$\int_C B \cdot dl = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \Phi_E.$$

The magnetic field around a closed loop is proportional to the current flowing through the loop (Ampere's Law) and the change in electric flux through the loop. The first part - the current - is an expression of a fundamental link between electricity and magnetism: moving charges create magnetic fields (check out this thread). The second part is basically an analogue to the second Maxwell equation (think of a simple circuit with a capacitor; when it's charging, current doesn't flow across the capacitor but there are magnetic fields all the same).

So here are the entire set, differential and integral form, of Maxwell's equations. It's useful to make intuitive sense of them.

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \qquad\qquad \iint_S E \cdot dS = \frac{Q_{enc}}{\epsilon_0} \tag{23}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad\qquad \int_C E \cdot dl = -\frac{\partial}{\partial t}\Phi_B \tag{24}$$

$$\nabla \cdot B = 0 \qquad\qquad \iint_S B \cdot dS = 0 \tag{25}$$

$$\nabla \times B = \mu_0 J + \mu_0\epsilon_0\frac{\partial E}{\partial t} \qquad\qquad \int_C B \cdot dl = \mu_0 I_{enc} + \mu_0\epsilon_0\frac{\partial}{\partial t}\Phi_E \tag{26}$$

2.2 Electrostatics

Electrostatics consist mostly of liberal applications of Gauss' Law. Consequently, many problems require the application of a "Gaussian surface" and solving for charge, potential, or field distributions.

As we know,

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \implies \iint_S E \cdot dS = \frac{Q_{enc}}{\epsilon_0}.$$

Using Gaussian surfaces, we can easily obtain the field of a sheet of charge, a line of charge, and a point charge, as a function of charge densities. These may be worth remembering, but know that these can be derived quickly (the non-ϵ Greek letters are the dimensionally-corresponding charge densities):

$$\text{sheet: } E = \frac{\sigma}{2\epsilon_0} \qquad \text{wire: } E = \frac{\rho}{2\pi\epsilon_0 r} \qquad \text{point: } E = \frac{q}{4\pi\epsilon_0 r^2}$$

We may describe the vector valued electric field with a scalar-valued potential. Consequently, integrating the field along a line gives the potential change.

$$E = -\nabla V \implies V = -\int E \cdot dl. \tag{27}$$

It's useful to know that V is *always* continuous!

Boundary conditions of electric fields at boundaries can be derived by picturing Gaussian pill-boxes and integrating the field around a loop. For a surface, the parallel field to the surface is the same inside and out:

$$E_{out}^{\parallel} = E_{in}^{\parallel} \tag{28}$$

while the perpendicular field is different depending on the surface charge density σ:

$$E_{out}^{\perp} - E_{in}^{\perp} = \frac{\sigma}{\epsilon_0}. \tag{29}$$

The work required for a certain charge configuration is

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i V_i, \tag{30}$$

which is one-half the sum of the charges, weighted by the potential at the position of each charge (from all the other charges).

Electric fields store energy, given by

$$U_E = \frac{\epsilon_0}{2} \int |E|^2 \, dV. \tag{31}$$

The capacitance is defined as the charge over potential

$$C = \frac{Q}{V} \tag{32}$$

and for a parallel-plate capacitor with separation d and plate area A

$$C = \frac{\epsilon_0 A}{d}. \tag{33}$$

The energy of the field in a capacitor is

$$U_C = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C V^2. \tag{34}$$

2.3 Magnetostatics

Analogously, magnetostatics consist mostly of liberal applications of Ampere's Law:

$$\nabla \times B = \mu_0 J \implies \int_C B \cdot dl = \mu_0 I_{enc}.$$

Like with Gauss' Law, we can obtain the magnetic field of various current configurations by thinking about Amperian loops. Most commonly encountered will be simple wires, current looping in a toroid, a simple loop, and a solenoid. Again, perhaps worth remembering, or derive them if you're fast. Don't forget to use the right hand rule to figure out their directions!

$$\text{wire: } B = \frac{\mu_0 I}{2\pi r} \qquad \text{toroid: } B = \frac{I N \mu_0}{2\pi r} \qquad \text{solenoid: } B = \mu_0 n I \qquad \text{loop: } B = \frac{\mu_0 I}{2r}.$$

For the toriod, r is the distance from the center to the point inside the toroid where we want the field. In general, N refers to the number of turns in the wire, and n refers to the number *density* of turns in the wire.

Sometimes, though, there will be a configuration that isn't so symmetrical or nice. We will then need to apply the Biot-Savart law for a wire carrying current I:

$$B(r) = \frac{\mu_0 I}{4\pi} \int \frac{d\left(l \times r'\right)}{r'^2}. \tag{35}$$

The Lorentz force on a charged particle is

$$F = q\left(E + v \times B\right). \tag{36}$$

Just like with electric fields, we have boundary conditions. These can be derived by thinking about Gaussian pillboxes (and applying $\nabla \cdot B = 0$) and Amperian loops that straddle the surface:

$$B_{out}^{\perp} = B_{in}^{\perp} \tag{37}$$

and with a surface current density K, surface normal n,

$$B_{out}^{\parallel} - B_{in}^{\parallel} = \mu_0 K \times n. \tag{38}$$

A particle in a field may exhibit cyclotron behavior, equating centripetal force with Lorentz force:

$$\frac{mv^2}{R} = qvB \tag{39}$$

from which we can derive the cyclotron frequency ω and the radius R:

$$\omega = \frac{v}{R} = \frac{qB}{m} \qquad R = \frac{mv}{qB}. \tag{40}$$

Magnetic fields also store energy:

$$U_B = \frac{1}{2\mu_0} \int |B|^2 \, dV. \tag{41}$$

2.4 Dynamics

Just be familiar with applying Faraday's Law:

$$\int_C E \cdot dl = EMF = -\frac{\partial \Phi_B}{\partial t}. \tag{42}$$

Many of the questions on this topic will be purely qualitative. Often, the GRE will ask for directions of current or motion.

There's also something called self-inductance for a solenoid, which is when a magnetic field is produced from the current in the wires. We define a constant of proportionality called the inductance L:

$$\Phi_B = LI. \tag{43}$$

2.5 Dipoles

A dipole is a vector quantity. For electric dipoles, the simplest case is a configuration of two opposite charges q, with $\vec{d}$ pointing from the *negative* to *positive* charge (this is the opposite direction of the field!). Then, the dipole moment is

$$\vec{p} = q\vec{d}. \tag{44}$$

The potential of a dipole at $\vec{r}$ scales one inverse power more than for a point charge (which has $V \propto r^{-1}$), so we have the following scalings:

$$V(\vec{r}) \propto \frac{\vec{p} \cdot \hat{r}}{r^2} \quad \Longrightarrow \quad E(\vec{r}) \propto \frac{1}{r^3}. \tag{45}$$

A dipole in a field E will experience torque

$$\tau = \vec{p} \times E \tag{46}$$

and have potential energy

$$U = -\vec{p} \cdot E. \tag{47}$$

Magnetic dipoles are closely analogous, but because there are no magnetic charges, it is generated by a loop of current I. If $\vec{A}$ is the normal to the disc generated by the current loop, then the dipole moment is

$$\vec{m} = I\vec{A}. \tag{48}$$

The field can easily be found by the B-S Law or with Amperian loops - see previous sections!

Analogously to the electric case, the torque in a field is

$$\tau = \vec{m} \times B \tag{49}$$

and the energy is

$$U = -\vec{m} \cdot B. \tag{50}$$

2.6 Materials

For a volume of material, we can define a electric dipole moment per unit volume $\vec{P}$.

This will generate a surface charge density

$$\sigma = \vec{P} \cdot n \qquad (51)$$

where n is the surface normal and a volume charge density

$$\rho = -\nabla \cdot \vec{P}. \qquad (52)$$

Dielectrics are quite simple on the GRE. If you ever see a question regarding dielectrics, make the substitution

$$\epsilon_0 \to \kappa\epsilon_0$$

where κ is the dielectric constant, and that should be enough. A common application is that a dielectric increases the capacitance of a capacitor.

2.7 Radiation

EM radiation is composed of orthogonal oscillating electric and magnetic fields. This is the solution to a wave equation in free space, that can be derived from Maxwell's equations.
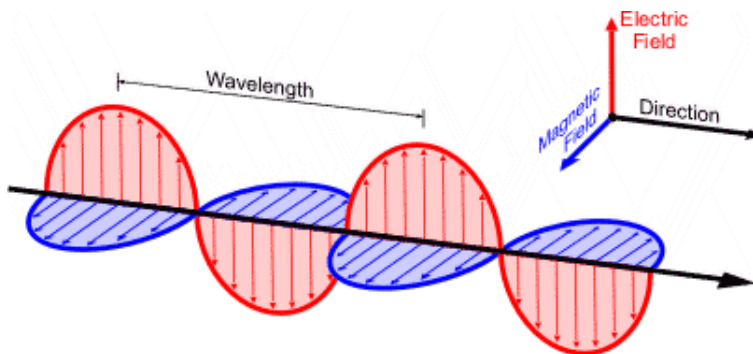
The direction that the wave propagates is denoted by the wavevector $\vec{k}$. The E field points in $\vec{n}$, which is orthogonal, and the B field points in $\vec{k} \times \vec{n}$.

The power radiated by a point charge q accelerating at a is

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} \qquad (53)$$

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but the constants won't matter - just remember $P \propto q^2 a^2$.

Oscillating dipoles also radiate. The total power radiated by a electric/magnetic dipole p/m oscillating with angular frequency ω are

$$P \propto p^2 \omega^4 \qquad P \propto m^2 \omega^4. \tag{54}$$

Again, no need to worry about useless constants. Additionally, the power of an oscillating electric dipole is many orders of magnitude greater than that of an oscillating magnetic dipole.

2.8   Circuits

Circuits are made of three main components:

- Resistors

    - $V = \frac{dQ}{dt}R = IR$
    - $P = IV = V^2/R = I^2 R$
    - resistances add linearly in series

- Capacitors

    - $V = \frac{Q}{C}$
    - $U = \frac{1}{2}CV^2$
    - capacitances add linearly in parallel

- Inductors

    - $V = L\frac{d^2 Q}{dt^2} = L\frac{dI}{dt}$
    - $U = \frac{1}{2}LI^2$
    - inductances add linearly in series

Given a circuit, writing down the sum of the voltages will lead to differential equations in Q, since all three components feature Q or its derivative(s). There are a few common types of circuits.

A RL, or resistor-inductor circuit, has a differential equation of the form

$$V_0 = IR + L\frac{dI}{dt},$$

with solutions of the form $I(t) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$ with a characteristic time constant $\tau = \frac{L}{R}$ (important to know).

A RC, or resistor-capacitor circuit, has instead

$$V_0 = \frac{dQ}{dt}R + \frac{Q}{C},$$

with similar solutions $Q(t) = V_0 C\left(1 - e^{-t/\tau}\right)$ and $\tau = RC$.

Finally, a RLC circuit, with a resistor-inductor-capacitor setup, has

$$V_0 = L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q$$

which behaves identically like a damped harmonic oscillator. The solutions for $Q(t)$ will also have under, over, and critically damped cases. Recalling section 1.5, in this case we have

$$\beta = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}.$$

Otherwise, everything else is exactly the same as in section 1.5.

3 Optics & Waves

This section is closely related to electromagnetism, but is more fundamental in certain ways. Like with electromagnetism, questions won't necessarily be conceptually straightforward. Spend time being intimately familiar with the concepts, and apply the correct relations.

3.1 Wave Basics

There is the basic wave equation
$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$
a statement relating the second time derivative to the second spatial derivative. It's also linear; ie, if f is a solution and g is a solution, then $f + g$ is also a solution.

The general solution is

$$f(x, t) = A \cos(kx - \omega t + \delta)$$

where δ is the phase, k is the wavenumber, related to the wavelength

$$\lambda = \frac{2\pi}{k} \tag{55}$$

and ω is the angular frequency, related to the period

$$T = \frac{2\pi}{\omega}. \tag{56}$$

Sometimes, a collection of wavelengths will be together and form a wave packet; then, v will depend on λ (or equivalently, k). We use a dispersion relation to describe the wave packet:

$$\omega(k) = v(k)k. \tag{57}$$

Then, we can define the *phase velocity*

$$v_p = \frac{\omega}{k} \tag{58}$$

and the *group velocity*

$$v_g = \frac{d\omega}{dk}. \tag{59}$$

The phase velocity is the speed of the moving waves within the wave packet; the group velocity is the travelling speed of the whole wave packet, and is the speed of information transferred by the wave.

3.2   Refraction

We define the index of refraction n, which depends on the medium; the speed in that medium is c/n.

The equation

$$\frac{\omega}{k} = \frac{c}{n} \tag{60}$$

will hold. Note that the frequency of the light depends on the source, not the medium, so as light changes medium, only the wavelength and speed will change.

3.3 Polarization

Polarization can be thought of as the direction of the electric field vector of a light wave. As such, we define a polarization vector $\vec{n}$ that points along the E field. Light can be transverse or longitudinally polarized. A polarizer is sort of like a filter that makes passing light polarized in one specific direction.

Malus' Law states that the intensity of outgoing light from a polarizer I is related to the intensity of incoming light via

$$I = I_0 \cos^2 \theta \tag{61}$$

where θ is the angle between the polarization vector of the incoming light and polarization vector of the polarizer. Therefore, one can deduce that unpolarized light incident on a polarizer will have half its intensity ouput:

$$I = \frac{I_0}{2}.$$

When light is reflected off a surface, it may also get polarized in the direction parallel to the surface. There is a critical angle of incidence where the light is completely polarized, called Brewster's Angle, given by

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) \tag{62}$$

where the light in medium n_1 reflects off a surface of medium with n_2.
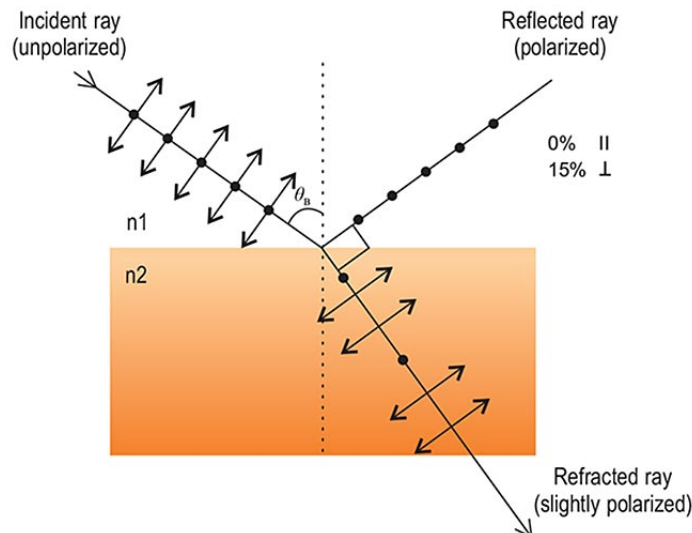
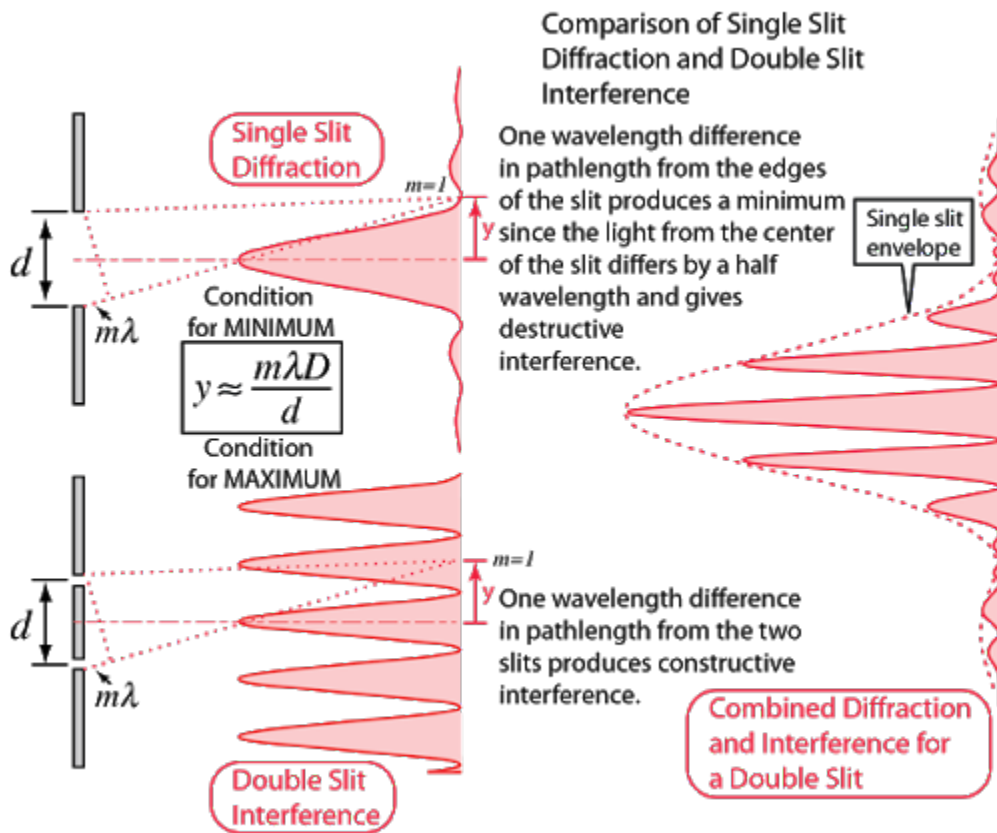## 3.4 Diffraction

The *maximum* (bright) fringes from double slit diffraction are described by

$$d \sin \theta = m\lambda \qquad (63)$$

where d is the slit spacing, θ is the angle from the central axis to a bright fringe, and m is any integer. If we want the distance y between fringes, we take $y = D \sin \theta$ and rearrange to get $y = \frac{m\lambda D}{d}$.

The *minimum* (dark) fringes from single-slit diffraction are given by the exact same formula. One critical difference is that there is no central dark fringe - so the case $m = 0$ does not exist.


Light and other quantum particles diffracting were important demonstrations of the wave nature of matter.

Bragg diffraction occurs in waves reflected off crystals, approximated as a lattice of atoms with spacing d. Diffraction fringes also occur in the reflected waves, and the maximum fringes are

given by

$$d \sin \theta = n \frac{\lambda}{2} \tag{64}$$

where θ is the angle of incidence, n is an integer, and λ is the wavelength. Confusingly, θ is not the same angle of incidence as we used for Brewster's angle before. See diagram below.
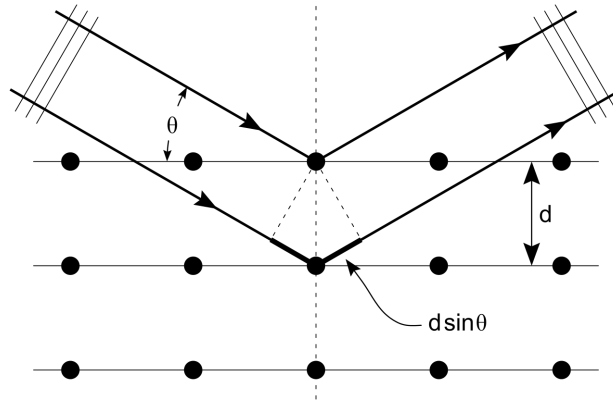
The Rayleigh Criterion describes the minimal angular separation for two objects viewed through a circular aperture to be resolved:

$$D \sin \theta = 1.22\lambda. \tag{65}$$

D is the aperture diameter, θ is the angular separation, and λ is the wavelength of the light. Just remember the constant 1.22.

3.5 Matter

The effective path length Δx of light in a medium with index of refraction n and physical length d is

$$\Delta x = nd. \tag{66}$$

The physical length is the length you get if you measured it with a ruler; the effective path length is the length that the light experiences, which is relevant to figure out superposition, multiples of wavelengths, etc.

Relevant for thin film questions, often light will be travelling from n_1 to n_2. If $n_1 < n_2$, reflected light gets a π phase shift; if $n_1 > n_2$, it gets no phase shift.

Rayleigh scattering is when light of wavelength λ scatters off atoms/particles of size a, with $\lambda >> a$. The scattered intensity is

$$I \propto \frac{I_0 a^6}{\lambda^4}.$$ (67)

This means that shorter wavelengths scatter more than longer ones (why the sky appears blue sometimes).

3.6   Geometric Optics

Refracting light between media n_1 and n_2 will obey Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$ (68)

Mirrors/lenses questions are also common; there are two formulas, but the tricky part is to apply them correctly.

We consider an object, and its image formed by four types of instruments: a concave mirror, a convex mirror, a concave lens, and a convex lens.

We have 5 main variables; the most common question is to be given some of them, and asked to solve for another.

- s, the object distance between the object and lens/mirror. It is always considered positive (apart from some multi-mirror/lens configurations).

- s', the image distance between the image and lens/mirror.

- f, the focal length, a characteristic of the lens/mirror. It is always positive for concave mirrors and convex lenses, and negative for convex mirrors and concave lenses.

- h, the object height. It is always positive.

- h', the image height.

These will obey

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ (69)

and

$$m = \frac{h'}{h} = -\frac{s'}{s} \tag{70}$$

where m is a quantity called the *magnification*.

Finally, some terminology: an image is *real* if $s' > 0$, and *virtual* if $s' < 0$. The object is *upright* if $h' > 0$, and *upside-down* if $h' < 0$.

Unless you're a genius, I highly recommend at least a few practice problems to really understand how to solve mirror/lens questions.

4   Thermodynamics & Statistical Mechanics

There's a lot of vocabulary, which unfortunately sometimes has to be memorized. Correspondingly, questions will often be focussed on concepts, utilising a lot of the fancy vocabulary. I found the calculations to be of minimal difficulty, but were often not conceptually accessible!

4.1   Stat Mech Basics

Statistical mechanics tries to produce macroscopic physics by analyzing microscopic physics.

A *system* is a bunch of matter composed of particles.

An *ensemble* is all the possible microstates corresponding to a macrostate. A macrostate can be described by state variables like number of particles N, the volume V, the energy E, the temperature T, etc.

The *canonical ensemble* refers to a system with fixed N, V, and T. The *microcanonical ensemble* by contrast, fixes N, V, and E.

If a system has discrete energy levels E_j, then the probability of the system being in the state with E_j is

$$p_j = \frac{e^{-\beta E_j}}{\sum_j e^{-\beta E_j}} \tag{71}$$

where $\beta = \frac{1}{k_B T}$, and $Z = \sum_j e^{-\beta E_j}$ is known as the partition function.

Knowing Z means you can obtain all the state variables of the system; for example, $\langle E \rangle = -\frac{\partial}{\partial \beta} (\log Z)$.

The equipartition theorem gives the internal energy U of a system: each quadratic term in the Hamiltonian contributes $\frac{1}{2}k_B T$ to the internal energy of the particle.

For example, the internal energy of a diatomic idea gas is $7 \cdot \frac{1}{2}k_B T$; 3 for translation, 2 for rotation, and 2 for the kinetic and potential energy of the spring-like bond.

4.2 Entropy

The entropy of a system is

$$S = k_B \log \Omega \tag{72}$$

where Ω is the number of microstates.

It can also be defined as

$$S = \frac{\partial}{\partial T}\left(k_B T \log Z\right). \tag{73}$$

Technically, these two cases apply to different ensembles, but this subtlety is not emphasized on the GRE - it's probably safe to assume both expressions for S hold.

Also useful is the entropy of an ideal gas, where

$$S \propto N k_B \log\left[\frac{V T^{3/2}}{N}\right]. \tag{74}$$

There are some other constants, but they are unimportant - just know the scaling.

The formula for combination might be useful:

$$nCm = \frac{n!}{(n-m)!m!} \tag{75}$$

as well as Stirling's Approximation:

$$\log\left(n!\right) \approx n \log n - n. \tag{76}$$

4.3 Thermodynamics Basics

In contrast to statistical mechanics, thermodynamics finds laws for macroscopic behavior with little regard to microscopic physics.

Here are the 4 laws:

1. Energy is conserved; $\Delta U = Q - W$.

2. If isolated system, heat is never transferred from cold to hot reservoirs; AKA "refrigeration is not spontaneous," or "heat engines always waste some heat," or "entropy of isolated systems cannot decrease," or $\Delta S \geq \int (\delta Q/T)$.

3. Entropy is zero at absolute zero.

4. (Zeroth Law) If two systems are both at thermal equilibrium with a third system, then they are at thermal equilibrium with each other.

4.4 Thermodynamic Relations

The fundamental identity:
$$dU = TdS - PdV. \tag{77}$$

From this, we can come up with useful relations like

$$T = \left.\frac{\partial P}{\partial S}\right|_V \qquad P = -\left.\frac{\partial U}{\partial V}\right|_S.$$

The Ideal Gas Law:
$$PV = Nk_BT. \tag{78}$$

Heat capacities are defined as

$$C_V = \left.\frac{\partial Q}{\partial T}\right|_V \qquad C_P = \left.\frac{\partial Q}{\partial T}\right|_P. \tag{79}$$

For C_V, for an ideal gas, if P is constant then $Q = U$. So then $C_V = \partial U/\partial T$ - we may then find U with the equipartition theorem.

Note that C_P and C_V are extensive properties that depend on the number of particles N. Experimentally, we may measure a (material-dependent) specific heat capacity c, defined in $Q = mc\Delta T$, where m is the mass.

4.5 Thermodynamic Processes

- A *quasistatic* process means that changes happen in infinitesimally small steps, usually very slowly, in equilibrium with the surroundings.

- A *reversible* process is a quasistatic process that can occur in reverse.

- An *adiabatic* process exchanges no heat with the surroundings: $\delta Q = 0$.

    - Adiabatic processes are reversible, if and only if $\Delta S_{system} = 0$. For an ideal gas, PV^γ is always constant, with $\gamma = C_P/C_V$.

    - Some adiabatic processes are not reversible. Then, $\Delta S_{system} \neq 0$.

- An *isobaric* process is constant in pressure. For ideal gases, $W = P\Delta V$.

- An *isothermal* process is constant in temperature. For ideal gases, $W = Nk_BT \int \frac{dV}{V}$.

- An *isochoric* process is constant in volume. For ideal gases, $W = 0$.

Systems and heat engines are just cycles made of thermodynamic processes. Macroscopically, Q_H enters the system from a hot reservoir at T_H, and Q_C leaves the system to a cold reservoir Q_C at T_C, in the process doing work W. The efficiency is

$$e = 1 - \left| \frac{Q_C}{Q_H} \right| \tag{80}$$

and the maximum efficiency possible for a system is the Carnot efficiency (associated with the Carnot cycle)

$$e_{max} = 1 - \frac{T_C}{T_H}. \tag{81}$$

It is also useful and important to know how to solve P-V and T-S diagrams.

4.6 Quantum

Particles are fermions (half-integer spin) or bosons (integer spin) and are often in systems with multiple energy levels.

The Fermi-Dirac distribution describes the average number of fermions in energy level with energy ϵ_i:

$$F_{FD} = \frac{1}{\exp\left(\left(\epsilon_i - \mu\right) k_B T\right) + 1} \tag{82}$$

and analogously the Bose-Einstein describes bosons:

$$F_{BE} = \frac{1}{\exp\left(\left(\epsilon_i - \mu\right) k_B T\right) - 1}. \tag{83}$$

In both cases, μ is the chemical potential.

A Bose-Einstein condensate is roughly the state where $T \to 0$ and $F_{BE} \to \infty$.

4.7 Miscellaneous

The root-mean-square speed of particles in an ideal gas is

$$v_{rms} = \sqrt{\frac{3k_B T}{m}} \tag{84}$$

where m is the mass of the particle.

5 Quantum Mechanics & Atomic Physics

I find that (especially in contrast to how you probably learned it) quantum mechanics on the GRE requires much more recall and knowledge rather than derivation or calculation. If you are familiar with derivations and proofs in arriving at quantum mechanical conclusions, that will only help you - but it's also helpful to know common facts off the top of your head (like the energy of the ground state hydrogen electron) to speed things up on the GRE.

5.1 Notations

A state, which is a vector in Hilbert space, is represented by a ket $|a\rangle$. Its conjugate transpose is represented by a bra $\langle a|$. An inner product between two vectors is written as $\langle b|a\rangle$.

Operators can be understood as matrices and act on states (vectors). The expectation value of an operator A acting on state $|a\rangle$ is $\langle a| A |a\rangle$.

5.2 Equations

The only equation of motion you will need to worry about is the (1-d) Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

where $\hat{H}$ is the Hamiltonian "total energy" operator. The Schrödinger equation can be solved by guessing a solution of the form $\Psi = \psi(x)\phi(t)$. Then the Schrödinger equation becomes separable and we get a time and position solution.

The time solution is
$$\phi(t) = \exp\left(-\frac{iEt}{\hbar}\right).$$

The position equation (AKA the time-independent Schrödinger equation) is

$$\hat{H}\psi(x) = E\psi(x)$$

which is just the eigenvalue equation for the $\hat{H}$ operator. Therefore, its solutions ψ are associated with definite values of energy E, and the solutions will depend on the potential V (contained in $\hat{H}$).

Then, we can obtain a solution of the form

$$\Psi = \exp\left(-\frac{iEt}{\hbar}\right)\psi(x).$$

Due to linearity of the solutions, we may construct a general solution

$$\Psi = \sum_{n=1}^{\infty} c_n \psi_n \exp\left(-\frac{iE_n t}{\hbar}\right). \tag{85}$$

That is, the collection of ψ_n form an orthogonal eigenbasis for the Hilbert space.

5.3 Operators

The most important thing to know is the commutation relation for position and momentum:

$$[x, p] = i\hbar. \tag{86}$$

Recall that a nonzero commutation relation means that the order of operator application matters.

There's a useful general property

$$[AB, C] = A\,[B, C] + [A, C]\,B \qquad [A, BC] = [A, B]\,C + B\,[A, C]. \tag{87}$$

More on the Hamiltonian: if any operator commutes with the Hamiltonian, then it is conserved:

$$\left[\hat{H}, \hat{O}\right] = 0 \implies \hat{O} \text{ conserved} \tag{88}$$

the proof is not difficult, but pretty irrelevant for the GRE.

5.4 Common Simple Potentials

The quantum harmonic oscillator is the most common. The Hamiltonian is of the form $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. For this potential, the energy eigenvalues are evenly spaced, and associated energy eigenstates are denoted as $|n\rangle$ where n ranges from 0 to ∞. It's usually not helpful and clunky to write out the energy eigenstates as explicit functions of x.

The energies of the eigenstates are $\frac{\hbar\omega}{2}$, $\frac{3\hbar\omega}{2}$, $\frac{5\hbar\omega}{2}$, etc...

There exist the raising and lowering operators:

$$a^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle \qquad a|n\rangle = n\,|n-1\rangle. \tag{89}$$

The infinite square well has eigenstates and eigenvalues

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \qquad E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \tag{90}$$

where a is the width of the square well.

A free particle nominally has an eigenstate and eigenvalues of the form

$$\psi = e^{\pm ikx} \qquad E = \frac{\hbar^2 k^2}{2m} \tag{91}$$

where k is the wavenumber ($p = \hbar k$). The wavefunction is not normalizable, but if we permit a range of k, then we end up with a wave packet, and it becomes normalizable. This is one way to understand the uncertainty principle.

Since $E = \hbar\omega$, we have $\omega = \frac{\hbar k^2}{2m}$ giving us a dispersion relation.

A delta function potential, $V(x) = -A\delta(x)$ only has one bound state:

$$\psi = \frac{\sqrt{mA}}{\hbar} \exp\left(-\frac{mA|x|}{\hbar^2}\right) \qquad E = -\frac{mA^2}{2\hbar^2}. \tag{92}$$

Finite square wells/barriers are often coupled with scattering questions. For these, use the conservation of probability current and some common sense.

5.5   Three-Dimensional Mechanics

Importantly, note that $[x_i, p_j] = 0$ if $i \neq j$.

Most often you will see questions with central potentials, like an electron on a hydrogen atom. For central potentials, the wavefunction (solution to the 3-d Schrödinger equation) is separable

$$\psi = R(r)Y(\theta, \phi)$$

and that Y does not depend on the potential.

This is helpful because we can analyze the behavior of Y (spherical harmonics) and know that it applies to all central potential situations. The orbital angular momentum operators $L_x, L_y,$ L_z don't commute with each other, but they do commute with L^2. The collection of Y are eigenstates of both L^2 and L_z; we label Y by the associated eigenvalues:

$$L_z Y_l^m = m\hbar Y_l^m \qquad L^2 Y_l^m = l(l+1)\hbar^2 Y_l^m \qquad (93)$$

and l can only be integer values. Moreover,

$$m = -l, \; -l+1, \; \dots \; l-1, \; l \qquad (94)$$

and this makes sense, because the z component of the angular momentum can never be greater than the total. Orbital angular momentum is also quantized.

Spin angular momentum is similar; S_z and S^2 commute. Analogously,

$$S_z Z_s^{m_s} = m_s \hbar Z_s^{m_s} \qquad S^2 Z_s^{m_s} = s(s+1)\hbar^2 Z_s^{m_s} \qquad (95)$$

but the caveat is that s can only take on one fixed value, which can be an integer or half-integer, and is an intrinsic immutable property of the system. As before,

$$m_s = -s, \; -s+1, \; \dots \; s-1, \; s. \qquad (96)$$

Often, angular momentum states are labelled by the m, l, s, m_s quantum numbers alone.

When adding spin/orbital angular momentum,

$$m_{total} = m_1 + m_2 \qquad (97)$$

but

$$s_{total} = s_1 + s_2, \; s_1 + s_2 - 1, \; \dots \; |s_1 - s_2| \qquad (98)$$

Again, the z components behave predictably - and the total can be anywhere between their difference (most cancellation) and their sum (most constructive).

In chemistry, the notation is to label the l quantum number by letters: s, p, d, f for $l = 0, 1, 2, 3$ respectively.

5.6 Hydrogen

Hydrogen is a special case of three-dimensional quantum mechanics, along with some facts that must be remembered due to how ubiquitous hydrogen (and its GRE questions) is.

It has a central potential with a potential of the form

$$V(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r}$$

but that's not super important; the Bohr radius (most probable distance between the nucleus and electron) is

$$a = \frac{4\pi\epsilon_0 \hbar^2}{\mu q_p q_e} \tag{99}$$

where $\mu = m_e m_p / (m_e + m_p)$ is the reduced mass. The Bohr energy of the electron in the n-th level is

$$E_n = \frac{\hbar^2}{2\mu a^2}\frac{1}{n^2}. \tag{100}$$

Memorize that $E_1 \approx -13.6\ eV$.

The Bohr radius and energy levels may change depending on what "flavor" of hydrogen you're given. Common questions will ask you to analyze hydrogen with changes to mass and charge. For example, positronium, which is hydrogen with a positron instead of a proton, will have $\mu = m_e/2$, and consequently a is 2 times larger and E is 2 times smaller compared to normal hydrogen (which has $\mu \approx m_e$).

5.7 Perturbation

Given a slight perturbation of order λ to a Hamiltonian, what are the corresponding changes to the energy eigenvalues? If the perturbed Hamiltonian is $H = H_0 + \lambda H'$, then to first order energy shift is

$$E_n = E_n^0 + \lambda \left\langle \psi_n^0 \right| H' \left| \psi_n^0 \right\rangle \tag{101}$$

where E_n^0 and $|\psi_n^0\rangle$ are the energy eigenvalue and eigenstate associated with the original Hamiltonian H_0.

For second order, the correction adds a term

$$E_n = E_n^0 + \lambda \left\langle \psi_n^0 \right| H' \left| \psi_n^0 \right\rangle + \lambda^2 \sum_{m \neq n} \frac{|\langle \psi_m^0 | H' |\psi_n^0\rangle|^2}{E_n^0 - E_m^0}.$$

Technically there are corrections to the eigenstates too, but they're most likely irrelevant for the GRE.

Adiabatic theorem: provided that a system evolves slowly, if the system starts in an eigenstate of the initial Hamiltonian, it will end in the corresponding eigenstate of the final Hamiltonian.

5.8   Other Effects

For hydrogen, there are additional small effects (often explored via perturbation theory).

The *fine structure* of hydrogen arises when we consider the relativistic speed of the electron, and the spin-orbit ($S \cdot L$) of the electron; these shift certain energy levels. The order of this effect is around α^2 smaller than Bohr energies ($\alpha \approx \frac{1}{137}$ the fine structure constant).

The *Lamb shift* has to do with something something vacuum energy fluctuations, and is around a factor of α^3 smaller than Bohr energies.

Finally, *hyperfine splitting* comes from the spin-spin interaction between the spins of the electron and proton. The coupling splits the singlet and triplet ground state energies. It's the smallest effect, around a factor of $\alpha^2 \frac{1}{1000}$ smaller than Bohr energies. Photons making this small energy transition is the origin of the 21 cm line, a well-known astronomical observation.

5.9   Radiation Interactions

In light-atom interactions, the wavelengths of incident radiation are usually much larger than the size of the atom - assuming this is called the *electric dipole approximation*. To the atom, incident wavelengths look like uniform EM fields.

When using this approximation, certain electron transitions are "forbidden" (technically just very rare). This can be understood in terms of conserving angular momentum, and the photon having spin-1.

The *selection rules* ("permitted" transitions) only permit transitions where

$$\Delta m = 0 \text{ or } \pm 1 \tag{102}$$

$$\Delta l = \pm 1. \tag{103}$$

For example, a $2p$ ($l = 1$) to $1s$ ($l = 0$) is permitted, but $2s$ ($l = 0$) to $1s$ ($l = 0$) is forbidden. These rules are useful for predicting whether an electron transition will take place.

In general, radiation interacting with matter depends on the energy of radiation.

For radiation of energy E less than a few keV, the photoelectric effect is predominant. The maximum energy of liberated electrons is given by

$$E_{max} = E - \phi \tag{104}$$

where ϕ is the material-specific work function.

In the range of E between tens of keV and a few MeV, we mostly have Compton scattering. The wavelength shift of the photon is

$$\Delta\lambda = \frac{h}{mc}\left(1 - \cos\theta\right) \tag{105}$$

where m is the electron mass and θ is the scattering angle.

Finally, for energetic photons more than the $2m_e c^2$ threshold, pair production becomes predominant.

5.10 Blackbody Radiation

A blackbody is something that absorbs all incident radiation, so observed emissions are purely due to its temperature.

The total energy radiated per unit surface area per unit time over all wavelengths is

$$\frac{dP}{dA} \propto T^4. \tag{106}$$

Wien's Law states that the peak wavelength of a blackbody radiation spectrum depends solely on its temperature:

$$\lambda_{peak} = \left(2.9 \times 10^{-3}\ K \cdot m\right) T^{-1}. \tag{107}$$

6 Special Relativity

Like with kinematics, you will mostly get questions that require a superficial calculation or problem solving step. Relativity is trippy enough that I have a hard time solving and thinking about problems just in my head. There are a few common equations that, when applied correctly, are guaranteed to solve any question you face.

6.1 Lorentz Transforms

A frame s' is moving at v relative to s along their shared x axis. Then,

$$t' = \gamma \left(t - \frac{v}{c^2} x \right) \qquad x' = \gamma \left(x - vt \right) \qquad y' = y \qquad z' = z. \tag{108}$$

The inverse transform is the same thing, but with all the negative signs turned positive. Lorentz transforms are the default way to tackle relativity of simultanety questions.

6.2 Time and Length

Time slows down in a moving frame s'; a time interval $\Delta t'$ that passes in s' will pass as

$$\Delta t = \gamma \Delta t' \tag{109}$$

in the stationary frame s.

Length is contracted in a moving frame: a length L' measured in s' will appear as

$$L = \frac{L'}{\gamma} \tag{110}$$

in s. Whatever qualitiative questions the GRE does have on special relativity, it will likely be asking about length contraction or time dilation.

6.3 Miscellaneous

Addition of relativistic velocities: if frame s' is moving at speed v relative to frame s, and an object appears to be moving at speed u in frame s', then speed w of the moving object in frame s is

$$w = \frac{u + v}{1 + \frac{uv}{c^2}}. \tag{111}$$

Relativistic kinematics requiring the use of 4-vectors are unlikely to appear, but common results from working with 4-vectors are very useful (and often necessary). Most importantly,

$$E^2 = p^2 c^2 + m^2 c^4. \tag{112}$$

The spacetime interval is

A wavelength shift ratio caused by relativistic Doppler shift is given by

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} \tag{113}$$

where $\beta = v/c$. Remember that this is a shift caused by relativistic motion only. Don't confuse this with cosmological redshift, which is due to the expansion of the universe.

7   Miscellaneous Topics & Trivia

7.1   Error Propagation

If σ_f is the uncertainty (standard deviation) associated with quantity f, and f is a function of $x, y, z, ...$ and so on, where $x, y, z...$ are all independent (very common), then

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + ...} \tag{114}$$

and this gives the two familiar relations

$$f = x + y \implies \sigma_f = \sqrt{\sigma_x^2 + \sigma_y^2}$$

$$f = xy \implies \sigma_f = f\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}.$$

The *weighted average X* of quantities x and y is

$$X = \frac{\frac{x}{\sigma_x^2} + \frac{y}{\sigma_y^2}}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}} \qquad \sigma_X = \frac{1}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}}.$$
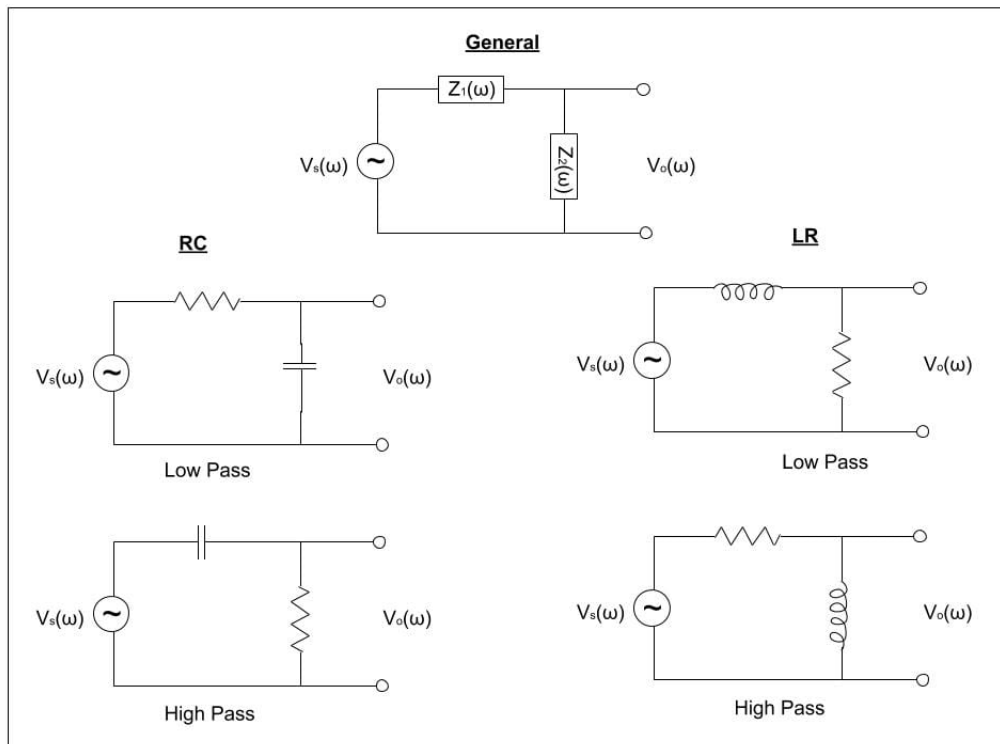
7.2   AC circuits

For AC circuits, imaginary numbers are useful for representing sinusoids.

We define the impedance Z which obeys Ohm's Law $V = IZ$. For common components,

$$\text{resistor:} Z = R$$

$$\text{capacitor:} Z = \frac{1}{i\omega C}$$

$$\text{inductor:} Z = i\omega L$$

where ω is the AC frequency.

With these components we can construct components like high and low pass filters:

To approach various filter setups, consider the impedances of various components, and think about how they behave as the frequency ω goes very large or small.

A sudden switch-on voltage can often be treated as $\omega \to \infty$.

7.3   Logic

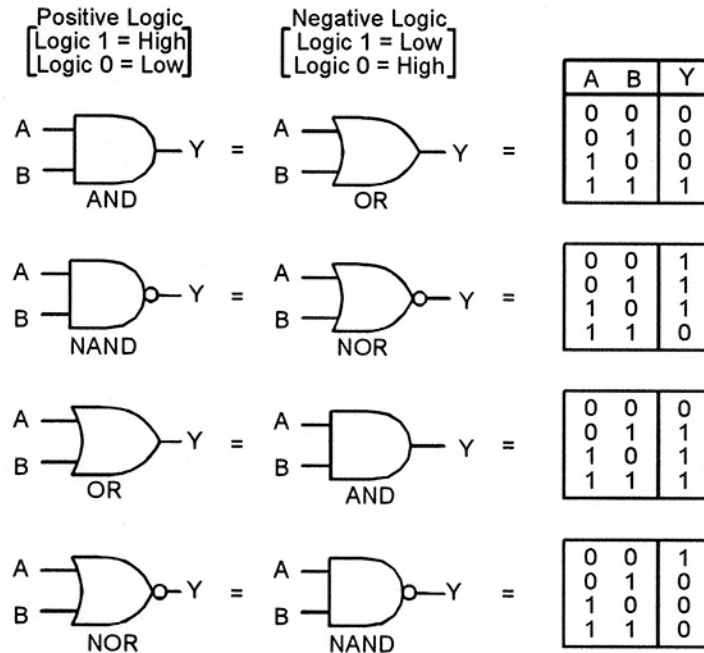Circuit logic on the GRE is very basic. Consider two binary inputs A and B.

An AND gate is represented as $A \cdot B$, while a OR gate is $A + B$.

Useful identities to construct NAND and NOR gates:

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$
$$\overline{A + B} = \overline{A} \cdot \overline{B}.$$

Summary:

It's perhaps time-consuming, but always foolproof, to construct a truth table for complicated arrangements of gates.

7.4 Condensed Matter

A *Fermi gas* is an ensemble of non-interacting fermions. We know from basic thermal physics that fermions obey the Pauli exclusion principle.

For a Fermi gas, the *Fermi energy* E_F is the energy level of the most energetic fermion, assuming that the whole system is in the ground state (this is also the chemical potential at absolute zero). In other words, if there are N particles in a ground state Fermi gas, E_F is the energy level below which there are N states.

In momentum space, the particles below E_F fill up a sphere of *Fermi radius* k_F (which we call the *Fermi surface*), and we define a *Fermi momentum*

$$p_F = \hbar k_F. \tag{115}$$

We may also express the Fermi energy as

$$E_F = \frac{\hbar^2 k_F^2}{2m}. \tag{116}$$

In reality, Fermi surfaces can get very complicated, affecting the behavior of the material/crystal in question. In a metal, for example, electrons form a delocalized "sea," and they can be modelled with a Fermi gas. If the electron number density is n, then the Fermi radius scales as

$$k_F \propto n^{1/3}.$$

7.5 Astrophysics

A central observation is that the universe is expanding. We can describe this with

$$d(t) = a(t)d_0,$$

where $d(t)$ is the proper distance between two astronomical objects, d_0 is this distance at some reference time t_0, and $a(t)$ is a dimensionless scale factor. By convention, t_0 is usually set to today, and so $a(t_0) = 1$.

The parameter $a(t)$ is of central importance in cosmology, and its behavior predicts the expansion rate of the universe.

After some simple derivations (which you can look up) we equivalently have Hubble's Law, which states that

$$v = Hd, \tag{117}$$

or that objects are moving away from Earth at a speed proportional to their distance, and $H = \dot{a}/a$.

Because of this cosmological expansion, we expect to measure cosmological redshift in radiation:

$$\frac{\lambda_{observed}}{\lambda_{emitted}} = \frac{a(t_0)}{a(t_{emitted})} \tag{118}$$

and we define the *redshift* z:

$$z = \frac{\lambda_{observed}}{\lambda_{emitted}} - 1. \tag{119}$$

Therefore we have $z = 0$ if we were to measure a source with no redshift (ie. radiation emitted nearby); if z is very large, we know that this would correspond to a very distant object.

To clarify, cosmological redshift is caused by *space itself* expanding. We observe another kind of relativistic redshift (Doppeler) from *relative motions* of astronomical objects. Questions on the GRE will be straightforward and probably will not induce confusion between the two.

7.6 Random Facts

The strong force produces particle interactions on the order of 10^{-23} seconds.

EM interactions are on the order of 10^{-18} to 10^{-16} seconds, a characteristic product being a photon.

Weak interaction are on the order of 10^{-10} to 10^{-8} seconds, a characteristic product being a neutrino.

———————————

Nuclear diameters are order femtometer sized (10^{-15} m).

———————————

Sneaky trick: if a (most likely nuclear) process has many decay channels that are in series, each with its distinct half-life, the total half-life τ is given by

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \dots$$

———————————

The Poisson distribution models the probability of a certain number of events n ocurring in a given time interval:

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!} \tag{120}$$

where λ is the expectation value of n.

———————————

There are usually some questions asking for general knowledge, like recent Nobel prizes, modern physics trivia, etc. It's hard to study for these; if you have time, I recommend skimming some textbooks. Wikipedia rabbit holes are also nice; I find that Wikipedia usually organizes physics and its subfields in a clear way and does a good job briefly describing results across different topics. The GRE will not ask questions that are in-depth - if you have even a superficial understanding of a certain topic it's usually enough to get a reasonable answer.

8   Logistical Comments

The physics GRE is a pretty disputed requirement. The general sentiment across academia appears to be that it's a pointless and pricey hurdle in the process to identify promising research students.

If you're a bit like me and have bothersome test anxiety, good news may be coming, since it appears that more and more schools seem to be abandoning or making optional the physics GRE. But in the case that this can't come soon enough, writing it will be just another hurdle to pass. Believe it or not, making it close to the end of an undergrad physics degree means that you're probably better at writing time-constrained tests on paper than you might think.

The test has 100 multiple choice questions to complete in 170 minutes. You get one point for a correct answer, and no point penalty for an incorrect one. Therefore, guess every question you don't know. If you're able to eliminate any answer choices, that already helps significantly!

Your raw score (out of 100) is then converted to a scaled score ranging from 390 to 990 depending on how everyone else performed. Usually, 82-85 questions correct or more will get you a perfect score of 990. Anything above 900 (70-80 questions correct usually) is pretty competitive, depending on who you ask.

As for actually attacking the exam, I only have one comment.

Time is your worst enemy! Many people can easily score perfect if they had more time - but because of the time constraint, you are forced to skip questions that you don't know how to solve. This can be quite unforgiving - for most questions, you must be able to identify how to solve it within seconds of looking at it, and immediately begin calculating the answer. This also leaves very little time, if any, to fix mistakes. Some questions are just trivia and take less time, leaving you more time to tackle the longer ones - but on average, 1.7 minutes per question is a rather tight limit. Keep a close eye on the clock during the test - I recommend checking every 5 or 10 questions. I have no systematic evidence for this, but for me personally the first half took comparatively less time (and felt much easier) than the second half. Around the 50 minute

mark, I was already on question 50 - but overall I still ended up skipping quite a few questions and really struggled to finish the second half. I experienced a similar trend on practice exams too.